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# Replica Symmetry Breaking in Renormalization: Application to the Randomly Pinned Planar Flux Array

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**Abstract.** — The randomly pinned planar flux line array is supposed to show a phase transition to a vortex glass phase at low temperatures. This transition has been examined by using a mapping onto a 2D  $XY$ -model with random anisotropy but without vortices and applying a renormalization group treatment to the replicated Hamiltonian based on the mapping to a Coulomb gas of vector charges. This renormalization group approach is extended by deriving renormalization group flow equations which take into account the possibility of a one-step replica symmetry breaking. It is shown that the renormalization group flow is unstable with respect to replica asymmetric perturbations and new fixed points with a broken replica symmetry are obtained. Approaching these fixed points the system can optimize its free energy contributions from fluctuations on large length scales; an optimal block size parameter  $m$  can be found. Correlation functions for the case of a broken replica symmetry can be calculated. We obtain both correlations diverging as  $\ln r$  and  $\ln^2 r$  depending on the choice of  $m$ .

## 1. Introduction

The technological aspects of high- $T_c$  superconductors in strong magnetic fields and especially of their ability to preserve superconductivity by flux pinning [1] have led to intense theoretical studies of the properties of a flux line array in a type-II superconductor with random point-like pinning centers [2, 5, 8–10, 12, 14, 16]. It has been conjectured [2–4] that the flux lines in a superconductor with point disorder form a new thermodynamic phase, the vortex glass phase. It is supposed that in this phase the flux lines are collectively pinned by the point defects and energy barriers between different metastable states of the flux line array occur which diverge with increasing length scale  $L$  leading to a glassy dynamics and zero linear resistivity [12, 15, 16]. But there is still too little conclusive evidence to confirm this scenario by analytic means.

Due to the absence of topological defects [15, 16] in  $1 + 1$  dimensions, the planar  $1 + 1$ -dimensional flux line array can be well treated analytically in an elastic approach. Actually, the system of flux lines in a type-II superconducting plane with parallel magnetic field and point disorder is the only system for which the existence of a vortex glass phase has been shown analytically [2, 5, 9, 10, 12] by applying various different methods.

On the other hand, the predictions for important physical features of the disordered 1 + 1 dimensional flux line array obtained by the different analytical methods differ significantly. In particular, there are still many competing conjectures concerning the correlations in the vortex glass phase. Moreover, the results of numerical simulations confirm neither of the analytical predictions [14]. Essentially three analytic approaches have been applied to the problem: (i) After using the replica trick and mapping onto a 2D  $XY$ -model with random anisotropy but without vortices [2, 5], a renormalization group (RG) calculation [6, 7] has been carried out with the replicated Hamiltonian not taking into account the possibility of replica symmetry breaking (RSB). (ii) The replicated Hamiltonian has also been studied by a variational treatment admitting of continuous RSB finding that a one-step breaking is realized [9, 10]. (iii) Without making use of the replica trick, the corresponding kinetic equation has been treated by a dynamical RG analysis [11, 12].

In the present paper we want to study how the concept of RSB could enter into a RG analysis. For this purpose we map the disordered planar flux line array onto the 2D  $XY$ -model with random anisotropy and perform a RG calculation with the replicated Hamiltonian where we generalize the set of coupling constants such that we can take into account a one-step RSB. Our aim is to show that an instability with respect to one-step RSB can also be found in the RG treatment. This leads to a more unified view of the approaches (i) and (ii), and results obtained by the variational approach can partly be reproduced in our calculation.

On scales larger than the flux line distance  $l$  the planar flux line array is described by an elastic model with the positions of the flux lines given by a scalar transverse displacement field  $\phi(\mathbf{r})l/\pi$ . The field  $\phi(\mathbf{r})$  itself can be regarded as a phase field giving the phase shift of the superconducting phase caused by the flux line displacements, because an increase in the flux line displacement by  $l$  induces a shift of  $\pi$  in the field  $\phi(\mathbf{r})$ . On large length scales, the planar flux line array interacting with the random pinning centers can then be described by the Hamiltonian [2, 4, 5, 17]

$$\frac{1}{T} \mathcal{H}[\phi] = \int d^2\mathbf{r} \left\{ \frac{1}{2} K (\nabla\phi)^2 + V_1(\mathbf{r}) \sin(2\phi(\mathbf{r})) + V_2(\mathbf{r}) \cos(2\phi(\mathbf{r})) \right\}. \quad (1)$$

The second term containing the random potential  $V(\mathbf{r})$  with zero average and short range correlations ( $i, j = 1, 2$ )

$$\overline{V_i(\mathbf{r})V_j(\mathbf{r}')} = 2g \delta_\xi(\mathbf{r} - \mathbf{r}')\delta_{ij} \quad (2)$$

models the interaction of the flux lines with the point disorder in the continuum description ( $g$  includes a factor  $1/T^2$ ). The function  $\delta_\xi$  is a delta-like function of the small width  $\xi$  given by the maximum of the flux line core radius and the impurity size. A crucial feature of this term is to respect the periodicity of the lattice, i.e. it is invariant under a uniform shift  $\pi$  of the field  $\phi(\mathbf{r})$ . By rescaling of one coordinate the isotropic first term for the elastic energy is obtained with one elastic constant  $K$  (including a factor  $1/T$ ).

Each of the three approaches sketched above [2, 5-7, 9-12] as well as the numerical simulation [14], yield a phase transition at  $K = 1/\pi$  or  $\tau = 0$  with

$$\tau = 1 - \frac{1}{\pi K}, \quad (3)$$

which serves as a small parameter measuring the distance from the transition and controlling expansions around the transition. For  $\tau < 0$  the system is in a high-temperature phase, disorder is not relevant and does not alter the correlations induced by thermal fluctuations on large length scales  $\overline{(\phi(\mathbf{r}) - \phi(\mathbf{0}))^2} = (\ln r)/(\pi K)$ .

However, the results concerning the correlations for  $\tau > 0$  in the glassy phase differ significantly. The RG analysis carried out on the replica Hamiltonian in a replica symmetric way yields correlations  $\langle (\phi(\mathbf{r}) - \phi(\mathbf{0}))^2 \rangle \propto \tau^2 \ln^2 r$  at large length scales  $r$  [6-8]. The same result is obtained in the dynamical RG calculation [11, 12]. Besides, correlations diverging like the square of the logarithm follow from a real-space RG procedure [13]. On the other hand, in the variational approach with one-step RSB, correlations  $\langle (\phi(\mathbf{r}) - \phi(\mathbf{0}))^2 \rangle = (\ln r)/(\pi K(1 - \tau))$  are found [9] to diverge logarithmically but with a prefactor increasing with decreasing temperature. Logarithmically diverging correlations have also been found in the numerical simulation [14] but the prefactor of the logarithm does not accord with the analytical prediction in [9]. Our calculation including one-step RSB in the RG analysis of the replicated Hamiltonian can reproduce both correlations diverging with a simple logarithm and correlations diverging with the square of the logarithm depending on the choice of the block size parameter  $m$  in the one-step RSB scheme.

## 2. RG Analysis

Introducing  $n$  replicas and averaging over the disorder gives the effective replicated Hamiltonian (with replica indices  $\alpha, \beta$  running from 1 to  $n$ )

$$\frac{1}{T} \mathcal{H}_R[\phi_\alpha] = \int d^2\mathbf{r} \left\{ \frac{1}{2} \sum_{\alpha\beta} K_{\alpha\beta} \nabla\phi_\alpha \cdot \nabla\phi_\beta - \sum_{\alpha\beta} g_{\alpha\beta} \cos(2(\phi_\alpha - \phi_\beta)) \right\} \quad (4)$$

with matrices  $K_{\alpha\beta}$  and  $g_{\alpha\beta}$  taking on their bare values

$$K_{\alpha\beta,0} = K\delta_{\alpha\beta} \quad (5)$$

$$g_{\alpha\beta,0} = g. \quad (6)$$

This Hamiltonian is equivalent to the replica Hamiltonian of a 2D XY-model with random anisotropy but without vortices [6]. The variational studies allowing for continuous RSB performed so far on this Hamiltonian [9, 10] have shown that in the 2-dimensional model considered here a one-step RSB is realized. Our calculation is restricted so far to a one-step RSB scheme but the results from the variational approach suggest that our results may stay valid even if it is possible to extend the calculation to higher steps of RSB or a continuous RSB scheme.

Introducing one-step RSB and following the resulting RG flow, it is necessary to admit matrices  $K_{\alpha\beta}$  and  $g_{\alpha\beta}$  of the form  $K_{\alpha\beta} = A\delta_{\alpha\beta} + B\tilde{\delta}_{\alpha\beta} + C$  and  $g_{\alpha\beta} = g_1\tilde{\delta}_{\alpha\beta} + g_2(1 - \tilde{\delta}_{\alpha\beta})$ ; the elements of the matrix  $\tilde{\delta}_{\alpha\beta}$  are 1 if  $\alpha$  and  $\beta$  belong to the same block of size  $m$  and 0 otherwise. Because of (5), (6) we have initially  $A_0 = K$ ,  $B_0 = C_0 = 0$  and  $g_{1,0} = g_{2,0} = g$ . To derivate the full RG flow equations, we perform an analysis technically similar to that of Cardy and Ostlund [6] but with significant extensions to take into account the one-step RSB. The calculation is based on the mapping onto a coupled Coulomb gas. We want to consider only weak disorder, so initially the disorder strength  $g$  will be small; also throughout the RG procedure the matrix elements of  $g_{\alpha\beta}$  stay sufficiently small to use standard methods [6, 18] to transform the cos-couplings in the partition sum and to integrate out the fields  $\phi_\alpha(\mathbf{r})$  in favour of integer charges  $n_{\alpha\beta}(\mathbf{r})$  ( $\alpha < \beta$ ) with fugacity  $g_{\alpha\beta}$ . The replicated disorder averaged partition sum  $\bar{Z}^n$  factors then into  $\bar{Z}^n = Z_{el} Z_C$  where the factor  $Z_{el}$  represents the purely elastic part of the partition sum; this factor plays a role only in deriving the RG equation for the free energy density and will be considered later on in detail. In the Coulomb gas factor  $Z_C$  of the partition sum, it has to be summed over all spatial configurations of interacting charges, which can take on any integer value, but because the charge fugacities  $g_{\alpha\beta}$  are sufficiently small, only positive

and negative unit charges have to be considered, which obey in addition a neutrality condition. Switching from the continuum description with a short wavelength cutoff  $\xi$  to a description on a square lattice with lattice constant  $\xi$  and lattice vectors  $\mathbf{R}$  for easier notation, one obtains the following expression for the replicated disorder averaged partition sum:

$$\overline{Z^n} = Z_{el} Z_C = Z_{el} \times \prod_{\mathbf{R}} \prod_{\alpha < \beta} \sum_{n_{\alpha\beta}(\mathbf{R})=-1}^1 \exp(-\mathcal{H}_C) \tag{7}$$

$$-\mathcal{H}_C = \frac{1}{2} \sum_{\mathbf{R} \neq \mathbf{R}'} \sum_{\alpha, \beta, \gamma, \delta} n_{\alpha\beta}(\mathbf{R}) (K^{-1})_{\beta\delta} n_{\gamma\delta}(\mathbf{R}') G'(\mathbf{R} - \mathbf{R}') + \sum_{\mathbf{R}} \sum_{\alpha\beta} (n_{\alpha\beta}(\mathbf{R}))^2 \ln g_{\alpha\beta}, \tag{8}$$

where  $G'(\mathbf{R}) = \ln(|\mathbf{R}|/\xi)$  and  $n_{\beta\alpha} = -n_{\alpha\beta}$  ( $\alpha < \beta$ ). The matrix  $(K^{-1})_{\alpha\beta} = a\delta_{\alpha\beta} + b\tilde{\delta}_{\alpha\beta} + c$  has the same block form as  $K_{\alpha\beta}$  with  $a = 1/A$ ,  $b = -B/A(A + mB)$  in the limit  $n \rightarrow 0$ .

The block form of the matrix  $g_{\alpha\beta}$  implies that two kinds of charges exist differing in their fugacities and, moreover, in their interactions with other charges due to the block form of the matrix  $(K^{-1})_{\alpha\beta}$ .

Taking this into account, a RG calculation in the style of Cardy and Ostlund (CO) [6] can be performed, which yields the following RG recursions in the limit  $n \rightarrow 0$  upon a change of scale by a factor  $e^l$  (Henceforth we always include a factor  $2/\pi$  in  $a$ ,  $b$  and  $c$  and a factor  $4\pi\xi^2$  in  $g_1, g_2$ ):

$$da/dl = -\frac{1}{8} a^2 m (g_1^2 - g_2^2) \tag{9}$$

$$db/dl = \frac{1}{8} a^2 (g_1^2 - g_2^2) \tag{10}$$

$$dc/dl = \frac{1}{8} a_0^2 g_2^2 \tag{11}$$

$$dg_1/dl = (2 - a)g_1 - \frac{1}{2}(2 - m)g_1^2 - \frac{1}{2}mg_2^2 \tag{12}$$

$$dg_2/dl = (2 - a - b)g_2 - mg_2^2 - (1 - m)g_1g_2 \tag{13}$$

The parameter  $m$  is a free parameter in these equations with  $0 \leq m \leq 1$  in the limit  $n \rightarrow 0$ ; possible choices for  $m$  will be discussed later on. (9), (10) show that

$$a + mb = a_0 = 2 - 2\tau \tag{14}$$

is not renormalized; this result is exact to all orders in the  $g_i$  ( $i = 1, 2$ ) due to a statistical invariance under tilt [6, 8].

In the special replica symmetric cases  $m = 1$  and  $m = 0$ , we get back the flow equations of CO: In the RG equations for  $m = 1$  ( $m = 0$ ),  $g_2$  ( $g_1$ ) plays the role of the single disorder strength parameter in [6], the diagonal matrix elements  $a + b$  ( $a$ ) of  $K_{\alpha\beta}^{-1}$  are not renormalized, and also the off-diagonal matrix elements  $c$  ( $b + c$ ) and the fugacity  $g_2$  ( $g_1$ ) renormalize as in [6]. The system exhibits the known CO fixed points  $g_2^* = 0$  ( $g_1^* = 0$ ) and  $g_2^* = 2\tau$  ( $g_1^* = 2\tau$ ). For  $m = 1$  the RG flow is sketched in Figure 1a.  $g_1$  ( $g_2$ ) does not feed back into the RG flow of the other quantities and does therefore not enter physical results like correlation functions (see below). For this reason the introduction of a small initial replica asymmetric perturbation  $\Delta g_0 = g_{1,0} - g_{2,0}$  has no effect on physical results if  $m = 1$  ( $m = 0$ ), although  $\Delta g$  turns out to be a relevant perturbation under RG (see below).

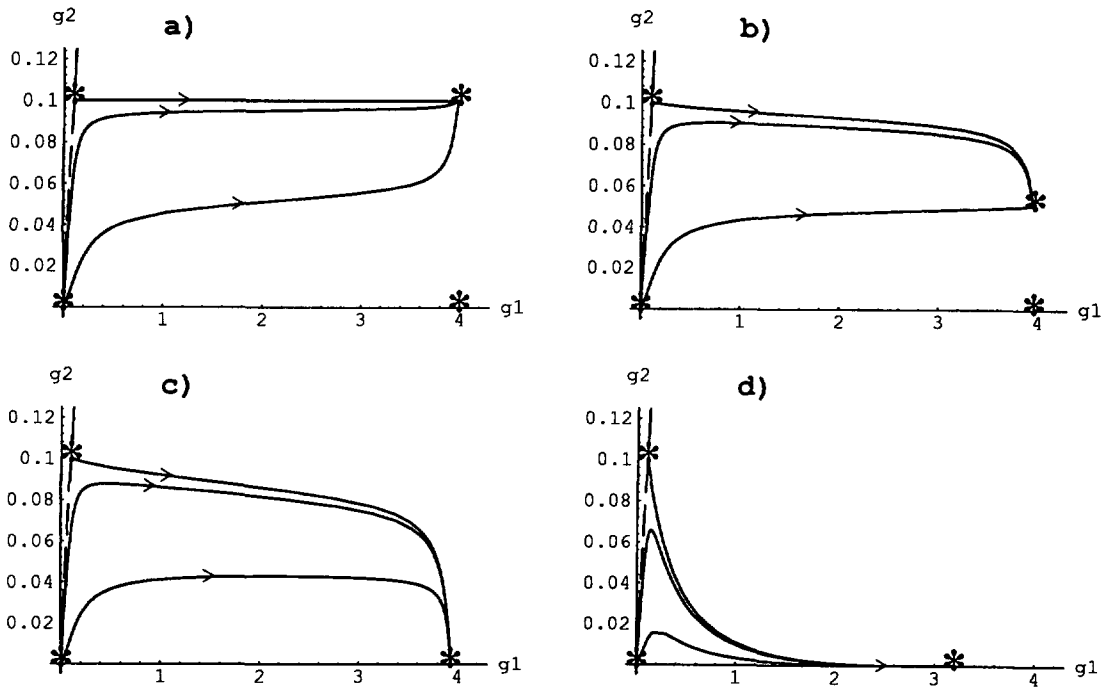


Fig. 1. — RG flow trajectories for  $\tau = 0.05$  and different values of  $m$  with an initial replica asymmetry  $\Delta g_0 = g_{1,0} - g_{2,0} = \tau^2$ ; a)  $m = 1$ , b)  $m = (1 + m^*)/2$ , c)  $m = m^*$ , d)  $m = 1 - 5\tau$ . The dashed line is the line  $g_1 = g_2$  of replica symmetric values. The trivial fixed point and the CO fixed point on this line as well as the RSB fixed points (17), (18) are plotted.

Starting as in (6) with replica symmetric initial conditions  $g_{1,0} = g_{2,0}$ , replica symmetry is preserved throughout the RG procedure independently of  $m$ , and  $a, b$  are not renormalized; therefore the CO scenario with the trivial fixed point  $g_1^* = g_2^* = 0$  and the non-trivial CO fixed point  $g_1^* = g_2^* = 2\tau$  is reproduced if replica symmetry holds initially.

However, introducing a small initial replica asymmetry  $\Delta g_0 = g_{1,0} - g_{2,0} \neq 0$  contrary to (6), the RG flow develops for  $\tau > 0$  an instability with respect to RSB. The system flows for  $\Delta g_0 > 0$  to a regime with  $g_1 > g_2$  and for  $\Delta g_0 < 0$  to a regime  $g_1 < g_2$  (entering on large length scales the unphysical regime of negative fugacities  $g_1$ ). In particular the replica symmetric CO fixed point  $g_1^* = g_2^* = 2\tau, a^* = a_0$  is unstable against small replica asymmetric perturbations. A linear stability analysis of the CO fixed point yields ( $\Delta a = a_0 - a$ )

$$d\Delta a/dl = \frac{1}{2} m a_0^2 \tau \Delta g \tag{15}$$

$$d\Delta g/dl = \frac{1}{m} \tau \Delta a. \tag{16}$$

These equations describe an instability with eigenvalue  $2(1 - \tau)\tau$  of the CO fixed point with respect to perturbations  $\Delta g$ . To avoid entering the unphysical regime of negative fugacities, we consider only perturbations  $\Delta g_0 > 0$ . As it is seen from (15), (16), such a perturbation causes the charge interaction strength parameter  $a$  to decrease and the asymmetry  $\Delta g$  to increase; finally,  $a$  renormalizes to 0 following (9). This flow towards the fixed point  $a^* = 0$  implies that one non-interacting type of unit charge with fugacity  $g_1$  appear on large length scales.

Furthermore, we can find from (12), (13) two additional non-trivial RSB fixed points (17) and (18) with  $a^* = 0$ ,  $b^* = a_0/m$  for each of them:

$$\begin{aligned} g_1^* &= 2 - \frac{1-m}{m} a_0 + a_0 \left(1 - \frac{2}{m} + \frac{4}{a_0}\right)^{1/2}, \\ g_2^* &= 2 - \frac{2-m}{m} a_0 - \frac{1-m}{m} a_0 \left(1 - \frac{2}{m} + \frac{4}{a_0}\right)^{1/2} \end{aligned} \quad (17)$$

$$\text{and } g_1^* = \frac{4}{2-m}, \quad g_2^* = 0. \quad (18)$$

At a certain

$$m^* = 1 - \tau/3 + \mathcal{O}(\tau^2) \quad (19)$$

the fixed points (17) and (18) fall exactly together. Only for  $m^* \leq m < 1$  the fixed point (17) is in the physical regime  $g_2^* \geq 0$  of non-negative fugacities. Moreover, the fixed point (17) is in this range of  $m$  stable with respect to perturbations in  $g_1$  and  $g_2$  (getting marginal with respect to perturbations in  $g_2$  at  $m = m^*$  where it coincides with (18)), whereas the fixed point (18) is unstable with respect to perturbations  $g_2 > 0$ . Therefore the fixed point (17) is attractive for all RG trajectories starting with  $g_{1,0} > g_{2,0} > 0$  (as illustrated in Fig. 1b) while the fixed point (18) is attractive only for RG trajectories with  $g_{1,0} > g_{2,0} = 0$ . For  $0 < m < m^*$  (18) is the only RSB fixed point in the physical regime of non-negative fugacities  $g_2 \geq 0$ . It is in this range of  $m$  the attractive fixed point for all RG trajectories with  $g_{1,0} > g_{2,0} \geq 0$  (see Figs. 1c, 1d); furthermore, it is stable with respect to perturbations in  $g_1$  and  $g_2$ .

As pointed out in (5), (6), the proper initial values  $K_{\alpha\beta,0} = K\delta_{\alpha\beta}$  and  $g_{\alpha\beta,0} = g$  are replica symmetric with  $\Delta g_0 = 0$ . It remains unclear in this approach how the initial asymmetry  $\Delta g_0 > 0$  necessary for the development of an instability with respect to RSB can be obtained from physical reasons. One hint is given in the next section where it is shown by comparison with the replica symmetric CO flow for  $\Delta g_0 = 0$  that contributions to the free energy from large scale fluctuations can be optimized (which means maximized in the limit  $n \rightarrow 0$ ) if a small perturbation  $\Delta g_0 > 0$  is introduced.

In the high-temperature phase for  $\tau < 0$  the system flows to the stable trivial replica symmetric fixed point  $g_1^* = g_2^* = 0$  regardless of an initial asymmetry  $\Delta g_0 \neq 0$ . In this phase the trivial replica fixed point is stable with respect to the RSB perturbation  $\Delta g_0$  so that RSB cannot occur in the high-temperature phase as it is expected. For  $\tau = 0$  the trivial fixed point stays marginally stable.

### 3. Free Energy and RSB

We want to proceed with a discussion of energetic aspects of the instability in the RG flow upon introducing a replica asymmetric perturbation  $\Delta g_0 > 0$  in the low-temperature phase. This enables us to fix the so far undetermined block size parameter  $m$  if  $\Delta g_0 > 0$  and to compare the free energy in the RSB case with  $\Delta g_0 > 0$  with the free energy in the replica symmetric CO case  $\Delta g_0 = 0$ . The standard procedure to determine  $m$  is to maximize the free energy density per replica in the limit  $n \rightarrow 0$  with respect to the additional free parameter  $m$ .

In the RG approach it is possible to derive the RG flow equation for the free energy density and calculate the free energy by integrating the flow equation; moreover, one can separate the contributions to the free energy from fluctuations on different length scales examining the flow of the free energy. In the RG procedure of increasing the cutoff  $\xi$  to  $\xi e^{dt}$  in the partition sum and rescaling the scale, one collects contributions not renormalizing the coupling constants; these contributions enter the renormalization of the free energy. Such terms are generated both

in the factor  $Z_C$  by contributions from integrating out the charge configurations as described above and in the factor

$$Z_{el} = \exp \left( -\frac{1}{2}(L/2\pi\xi)^2 \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} d^2q \left\{ n \ln(\xi^2 q^2/2\pi) \right. \right. \\ \left. \left. + n \left[ \frac{1-m}{m} \ln\left(\frac{\pi}{2}a\right) - \frac{1}{m} \ln\left(\frac{\pi}{2}a_0\right) + \frac{\pi^2}{4}ca_0 \right] \right\} \right) \quad (20)$$

(in the limit  $n \rightarrow 0$  and with  $L$  denoting the linear dimension of the system) by contributions from increasing the cutoff  $\xi$  and adjusting the couplings  $a, c$  according to the flow equations (9), (11). Finally, one obtains in the limit  $n \rightarrow 0$  for the free energy density per replica (apart from an additive constant independent of  $m$ ) the RG recursion relation

$$df/dl = 2f + \frac{1}{nL^2} dZ/dl \\ = 2f - \frac{1}{16\pi\xi^2} (mg_2^2 + (1-m)g_1^2) - \frac{1}{2\xi^2} \left( \left[ -\frac{2(1-m)}{m} \ln(a_0/a) - 2 \ln\left(\frac{\pi}{2}a_0\right) - \frac{\pi^2}{2}a_0c \right] \right. \\ \left. + \left[ -\frac{1-m}{m} \frac{1}{a} \frac{da}{dl} \right] + \left[ \frac{\pi^2}{4}a_0 \frac{dc}{dl} \right] \right). \quad (21)$$

From this recursion relation the initial free energy density  $f_0$  can be obtained by following the flow:

$$f_0 = - \int_0^{\infty} dl e^{-2l} \frac{1}{nL^2} dZ/dl. \quad (22)$$

Using (21), (22) we want to study the contributions to the free energy from fluctuations on different length scales in the low-temperature phase for the flow to the RSB fixed points (17), (18) induced by a small initial perturbation  $\Delta g_0 > 0$  and, in comparison, for the replica symmetric flow to the CO fixed point starting with  $\Delta g_0 = 0$ . To keep calculations tractable, we choose initial conditions in the vicinity of the CO fixed point, i.e.  $g_{1,0} = 2\tau + \Delta g_0$ ,  $g_{2,0} = 2\tau$ , when examining the RSB case. For general initial conditions with  $\Delta g_0 > 0$  there is initially a flow towards the CO fixed point slightly perturbed by the small asymmetry  $\Delta g_0 > 0$ ; for the study of the energetic effects of the RSB instability in the flow, this essentially replica symmetric part of the RG flow should be negligible.

As a consequence of the factor  $e^{-2l}$  appearing in the integrand of (22), the main contribution to  $f_0$  comes from the short scales. Starting at  $g_{1,0} = 2\tau + \Delta g_0$ ,  $g_{2,0} = 2\tau$  and evaluating (22) straightforwardly to the leading order in  $\Delta g_0$ , one obtains a maximum of  $f_0$  at the replica symmetric  $m = 0$ . This is because the main contribution to  $-(dZ/dl)/nL^2$  comes on short scales from the term  $[-\pi^2 a_0 c/2]/2\xi^2$ . The  $m$ -dependent part of  $[-\pi^2 a_0 c/2]/2\xi^2$  can be approximated by means of a linear stability analysis of the flow equations (9)–(13) at the CO fixed point enlarging on (15), (16) as  $[\Delta g_0(\pi^2 a_0^3/16)(1-m)(\exp(2\tau l) - 1)]/2\xi^2$  with a maximum at  $m = 0$ .

On the other hand, the maximization of the asymptotic large scale contributions leads to a quite different result. Examining the large scale contributions, one has to investigate the asymptotics of the integrand in (22) and to maximize  $-(dZ/dl)/nL^2$  in the limit of  $l \rightarrow \infty$ . For this purpose it is necessary to derive the asymptotics of  $c$  which is determined by the flow equation (11). From (11) follows that for  $m < m^* \leq 1$   $c$  is asymptotically linear divergent with an asymptotics  $c(l) \sim l g_2^{*2}(1-\tau)^2/2$  where  $g_2^*$  is taken in the stable RSB fixed point (17), which is  $g_2^* \simeq 6(m-m^*)$  to a good approximation. However, we find in the regime  $0 < m \leq m^*$  from (11) a saturation of  $c$  to a value  $c^*$  because the stable RSB fixed point is in this regime

given by (18) with  $g_2^* = 0$ . To obtain an estimate for  $c^*$  one has to determine the characteristic scale  $l^*$  on which  $g_2$  renormalizes towards 0; a linear stability analysis of the flow equations (9), (12), (13) at the CO fixed point extending (15), (16) reveals that  $l^*$  can be approximated as  $l^* \simeq (\ln(4\tau/(1-m)\Delta g_0))/2\tau$ . From (11) it follows  $c^* \simeq \tau \ln(4\tau/(1-m)\Delta g_0)$  for the leading order contribution in  $\tau$ . Moreover, it is seen from the flow equation (9) that  $a(l) \sim 1/l$  on large scales. Using these results for  $c$  and  $a$ , one can verify easily from (21) that the most divergent contributions to  $-(dZ/dl)/nL^2$  come from  $[-2 \ln(a_0/a)(1-m)/m - \pi^2 a_0 c/2]/2\xi^2$  for large  $l$ . Maximization of these terms yields

$$m = m^* = 1 - \tau/3 + \mathcal{O}(\tau^2), \quad (23)$$

because the maximization of the second term restricts  $m$  to values  $0 < m \leq m^*$  to avoid the occurrence of the linear divergence in the regime  $m^* < m \leq 1$  and the maximization of the only logarithmically diverging first term singles out the greatest value  $m = m^*$  of the interval  $0 < m \leq m^*$ . (23) is in fairly good agreement with [9].

RSB is a large scale effect associated with the existence of diverging energy barriers generating metastable states. Therefore it seems to be more reasonable to consider only the large scale contributions to the free energy in (22) although the expression (22) for  $f_0$  is dominated by its short scale part. This is equivalent to considering the free energy of the renormalized but not rescaled Hamiltonian on large scales but discarding a constant energy shift depending on  $m$  which comes from short scales. This energy shift, which is essentially replica symmetric, may describe the free energy of single metastable states. In the presence of an initial asymmetric perturbation  $\Delta g_0 > 0$ , maximization of the large scale contributions to the free energy yields then a maximum at  $m = m^*$  as derived above.

Comparison of these large scale contributions for the flow to the RSB fixed point when ( $\Delta g_0 > 0$ ) and for the replica symmetric flow to the CO fixed point ( $\Delta g_0 = 0$ ) shows that this part of the free energy is greater in the RSB case. This is because the most divergent contribution to  $-(dZ/dl)/nL^2$  is in the replica symmetric case as in the RSB case given by  $[-\pi^2 a_0 c/2]/2\xi^2$  with  $c(l) \sim l g_2^{*2}(1-\tau)^2/2$  but the replica symmetric CO fixed point value  $2\tau$  for  $g_2^*$  is always greater than or equal to ( $m = 1$ ) the RSB fixed point values given by (17), (18). Therefore it is energetically favorable on large scales to break the replica symmetry by introducing a perturbation  $\Delta g_0 > 0$ . This energy gain can occur on scales larger than  $L^* = \xi \exp(l^*|_{m=m^*}) = \xi(4\tau/(1-m^*)\Delta g_0)^{1/2\tau} = \xi(12/\Delta g_0)^{1/2\tau}$ .

In the high-temperature phase the trivial fixed point is stable with respect to the introduction of a perturbation  $\Delta g_0 \neq 0$ . For this reason the large scale contributions to the free energy are the same as in the replica symmetric case. For the short scale contributions to the free energy the same argumentation applies as in the low-temperature phase leading to a maximum at the replica symmetric  $m = 1$  if  $\Delta g_0 \neq 0$ . So RSB is energetically not favorable in the high-temperature phase as it is expected.

#### 4. Correlations

The RG flow and fixed point structure changes significantly upon introducing an energetically favorable, replica asymmetric perturbation  $\Delta g_0 > 0$  in the low-temperature phase as outlined above. As well the behaviour of the  $\langle \phi\phi \rangle$ -correlations changes drastically depending on the value of  $m$ . The Fourier transformed correlations between replicas on large scales can be calculated by using a Gaussian approximation to the renormalized but not rescaled replica Hamiltonian, which yields for small  $q \sim e^{-l}/\xi$  [8]

$$\langle \phi_\alpha(\mathbf{q})\phi_\beta(-\mathbf{q}) \rangle = \frac{1}{q^2} (K^{-1})_{\alpha\beta}(l = \ln(\xi/q)), \quad (24)$$



so that the large scale correlations depend essentially only on the asymptotic RG flow of the matrix elements  $a$ ,  $b$  and  $c$ .

From expression (24) one can verify that the connected correlation function

$$\overline{\langle \phi(\mathbf{q})\phi(-\mathbf{q}) \rangle - \langle \phi(\mathbf{q}) \rangle \langle \phi(-\mathbf{q}) \rangle} = \lim_{n \rightarrow 0} 1/n \sum_{\alpha, \beta} \langle \phi_\alpha(\mathbf{q})\phi_\beta(-\mathbf{q}) \rangle = (1 - \tau)\pi/q^2 \quad (25)$$

does not change its form at the transition, independently from the introduction of a nonzero  $\Delta g_0$  due to the non-renormalization of  $a + mb = a_0$ , contrary to the  $\langle \phi\phi \rangle$ -correlations [9].

For the  $\langle \phi\phi \rangle$ -correlations (24) yields

$$\overline{\langle \phi(\mathbf{q})\phi(-\mathbf{q}) \rangle} = \langle \phi_\alpha(\mathbf{q})\phi_\alpha(-\mathbf{q}) \rangle = \frac{\pi}{2q^2} (a(l) + b(l) + c(l))|_{l=\ln(\xi/q)}. \quad (26)$$

In the high-temperature phase no RSB takes place, even if  $\Delta g_0 > 0$ , and there is essentially no renormalization of  $a$ ,  $b$  and  $c$ ; the connected  $\langle \phi\phi \rangle - \langle \phi \rangle \langle \phi \rangle$ -correlation function and the  $\langle \phi\phi \rangle$ -correlation function coincide then and  $\overline{\langle (\phi(\mathbf{r}) - \phi(\mathbf{0}))^2 \rangle} = (1 - \tau) \ln(r/\xi)$ .

In the low-temperature phase the asymptotics of  $c$ , which is determined by the flow equation (11), is of special interest because in the replica symmetric case, i.e. without a replica asymmetric perturbation ( $\Delta g_0 = 0$ ), the asymptotics  $c \sim 2(1 - \tau)^2 \tau^2 l$  diverging linearly gives correlations  $\overline{\langle (\phi(\mathbf{r}) - \phi(\mathbf{0}))^2 \rangle} \sim ((1 - \tau)^2 \tau^2 / 2) \ln^2(r/\xi)$  diverging with  $\ln^2 r$  [8].

In the RSB case with an initial  $\Delta g_0 > 0$ ,  $c$  has also a linear divergent asymptotics  $c(l) \sim l g_2^* (1 - \tau)^2 / 2$  for  $m^* < m \leq 1$  (see above), where  $g_2^* \simeq 6(m - m^*)$  is taken in the stable RSB fixed point (17). This entails  $\langle \phi\phi \rangle$ -correlations diverging also with  $\ln^2 r$  for this range of  $m$  but with a prefactor reduced by a factor  $9(m - m^*)^2 / \tau^2 < 1$  compared to the replica symmetric case; in particular, we get back the replica symmetric CO result choosing  $m = 1$ . The situation changes significantly in the regime  $0 < m \leq m^*$  because the stable RSB fixed point is in this regime given by (18) with  $g_2^* = 0$ . Therefore  $c$  saturates on large scales to a value  $c^* \simeq \tau \ln(4\tau / (1 - m)\Delta g_0)$  for the leading order contribution in  $\tau$  (see above) and we obtain from (26) only logarithmically divergent  $\langle \phi\phi \rangle$ -correlations with a prefactor  $a_0/2m + c^*/2$  which is greater than in the high-temperature phase. To the leading order in  $\tau$  this yields correlations  $\overline{\langle (\phi(\mathbf{r}) - \phi(\mathbf{0}))^2 \rangle} \simeq ((1 - \tau)/m + \tau(\ln(4\tau / (1 - m)\Delta g_0)) / 2) \ln(r/\xi)$ ; with our above choice (23) of  $m \simeq 1 - \tau/3$  we get  $\overline{\langle (\phi(\mathbf{r}) - \phi(\mathbf{0}))^2 \rangle} \simeq ((1 - 2\tau/3) + \tau(\ln(12/\Delta g_0)) / 2) \ln(r/\xi)$ . This implies that the prefactor of the logarithm increases with  $\tau$  in the low-temperature phase.

Our results for the low-temperature phase show that within the one-step RSB RG approach with a small initial replica asymmetry  $\Delta g_0 > 0$ , it is possible to obtain the known replica symmetric result for the  $\langle \phi\phi \rangle$ -correlations diverging like  $\ln^2 r$  [8] if  $m = 1$  as well as  $\langle \phi\phi \rangle$ -correlations diverging in the same way but with a smaller prefactor if  $m^* < m \leq 1$  and logarithmically divergent  $\langle \phi\phi \rangle$ -correlations with a prefactor increasing with decreasing temperature if  $0 < m \leq m^*$ . The latter possibilities are of interest with regard to the numerical simulations [14] and the results of the variational approaches [9, 10].

## 5. Conclusion

To summarize we have shown an instability in the RG flow of the disordered planar flux line array, which is equivalent to the 2D  $XY$ -model with random anisotropy but without vortices, with respect to a one-step RSB. The flow approaches new RSB fixed points if an initial replica asymmetric perturbation is introduced. The system can optimize its free energy contributions from large length scale ( $> L^*$ ) fluctuations by breaking the replica symmetry and approaching the RSB fixed point where the energetical optimal choice of  $m$  is  $m = m^* \simeq 1 - \tau/3$ . Introducing

the initial perturbation, the  $\overline{(\phi\phi)}$ -correlations show a  $\ln^2 r$ -divergence on large length scales in the range  $m^* < m \leq 1$  returning to the replica symmetric result at  $m = 1$ ; for  $0 < m \leq m^*$  the correlations diverge only as  $\ln r$ , which is especially for  $m = m^*$  the case.

During completion of this work P. Le Doussal and T. Giamarchi have submitted a letter [19] in which they find independently from our results an instability with respect to RSB in the 2D XY-model in a random field.

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