

Topological Order in the Phase Diagram for High-Temperature Superconductors with Point Defects

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Applying a Lindemann-like criterion derived by Kierfeld, Nattermann and Hwa (preprint cond-mat/9512101), we calculate the magnetic field and temperature for a high- T_c superconductor, at which a topologically ordered vortex glass phase becomes unstable with respect to a disorder-induced formation of dislocations. The employed criterion is shown to be equivalent to a conventional phenomenological Lindemann criterion including the values for the numerical factors, i.e., for the Lindemann-number. The positional correlation length of the topologically ordered vortex glass is calculated.

I. INTRODUCTION

The influence of disorder on the Abrikosov vortex lattice in the mixed phase of high-temperature superconductors, such as $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+x}$ (BSCCO), is an issue of immediate technological interest because pinning of the flux lines by disorder opens the possibility of regaining a dissipation-free current flow in the mixed phase. The flux line (FL) array in a high-temperature superconductor (HTSC) is extremely susceptible to thermal and disorder-induced fluctuations due to the interplay of several parameters, namely the high transition temperature T_c , large magnetic penetration depths λ and short coherence lengths ξ , and a strong anisotropy of the material. This leads to the existence of a variety of fluctuation dominated phases of the FL array and very rich phase diagrams for the HTSC materials [1]. We want to consider here the pinning of FLs by point defects such as the oxygen vacancies, which is usually referred to as point disorder. It is well-known that the FL lattice is unstable to point disorder [2]. It has been conjectured that due to a *collective pinning* by the point disorder, the FL array may form a *vortex glass phase* with zero linear resistivity [3–6]. Though the existence of a vortex glass (VG) phase has been verified experimentally [7–11], its large scale properties characterizing the nature of the VG phase are still under debate [1]. A possible scenario for a description of the low-temperature properties of the FL array subject to point disorder is the existence of a topologically ordered, i.e., *dislocation-free*, VG phase, the so-called *Bragg glass* phase [17] as a thermodynamically stable phase. In this glassy phase, a quasi long range positional order of the FL array is maintained in spite of the pinning [5,17]. This entails the existence of algebraically decaying Bragg peaks in diffraction experiments on this phase (bearing the name “Bragg glass” for this property), which have indeed been observed in neutron diffraction experiments on BSCCO at low magnetic fields [12]. In the Bragg glass phase, the disordered FL array is modeled as an elastic manifold in a periodic random potential, similar to a randomly pinned charge density wave or a XY model in a random field [14–17]. To give a thermodynamically stable phase, this requires the persistence of the topological order, or absence of unbound dislocations, even in the presence of disorder. In neutron diffraction experiments, it has been observed by Cubitt et al. [12] that upon increasing the magnetic field, the Bragg peaks vanish, indicating an instability of the Bragg glass phase. Critical current measurements of Khaykovich et al. [13] show a sharp drop in the (local) critical current j_c upon decreasing the magnetic induction below a critical value. This can be attributed to the “disentanglement” of FLs in the absence of dislocations when topological order is regained and dislocation loops vanish upon lowering the magnetic field. The existence of such a transition has also been demonstrated in recent numerical studies [20,21]. In the closely related 3D XY model in a random field, vortex loops occur in a topological phase transition at a critical strength of the random field [20]. In simulations of disordered FL arrays [21], a proliferation of dislocation lines has been found at a critical magnetic field in good agreement with the experimental results in Ref. [12].

Recently, the quantitative aspects of this issue have been addressed also analytically [18,19], and in Ref. [18], a Lindemann-like criterion has been derived describing the range of stability of the Bragg glass phase with respect to a spontaneous formation of dislocation loops:

$$R_l > c^{1/2\zeta} (l^2 + \lambda^2)^{1/2} \simeq c^{1/2\zeta} \max\{l, \lambda\}, \quad (1)$$

where l is the FL distance and λ the magnetic penetration depths (we consider a magnetic field perpendicular to the CuO-planes of the HTSC, and will specify λ below for such a geometry, see (4)). R_l is the (transversal) *positional correlation length* of the disordered FL array, which is defined as the crossover length to the asymptotic large scale behaviour of the Bragg glass phase, where the average FL displacement starts to exceed the FL spacing l . c is a number, which was obtained in Ref. [18] to be of the order of $c \approx \mathcal{O}(50)$, and $\zeta \approx 1/5$ is the roughness exponent

of the pre-asymptotic so-called “random manifold” regime, see (22) below. At the boundaries of the regime given by (1), a topological transition occurs, and dislocations proliferate; however, the nature of this transition could not be identified unambiguously in Ref. [18]. Beyond the transition line the FL array may form an amorphous VG with vanishing shear modulus or a viscous FL liquid.

The article is divided into three parts. First, we will review the pre-asymptotic regimes of the FL array subject to point disorder on scales smaller than the positional correlation length R_l . This allows us to express R_l in terms of the microscopic parameters of the HTSC and the disorder strength, and to obtain its dependence on magnetic induction B and temperature T . In the second part, we will demonstrate the equivalence of the above criterion (1) to the Lindemann criterion in its conventional form $u^2(l) < c_L^2 l^2$, where $u^2(l)$ is the (disorder-averaged) mean square displacement of the FLs on the *shortest scale*, the FL spacing l , and c_L is the Lindemann-number. This yields a relation $c \approx c_L^{-2}$ between c from (1) and the Lindemann-number c_L , and the value $c \approx \mathcal{O}(50)$ found by a variational calculation in Ref. [18] turns out to be in good agreement with a value $c_L \approx 0.15$ widely used in the literature for the Lindemann-number. This equivalence further supports a scenario where the topological transition of the FL array subject to point disorder may be described as disorder-induced melting by unbound dislocations on the shortest scale l [18]. Finally, and most importantly from the experimental point of view, we calculate the region of the phase diagram in the B-T plane (see Fig. 1) where the Bragg glass phase is stable and should be observable experimentally or numerically according to the Lindemann-like criterion (1). We find good agreement with the experiments [12,13]. The upper phase boundary of the Bragg glass, which we obtain using (1), turns out to be identical to the one obtained by Ertaş and Nelson [19]. They use a “cage model” to mimic the interactions between FLs which yields an effective theory for a single FL in a random potential, to which they apply the conventional phenomenological Lindemann criterion.

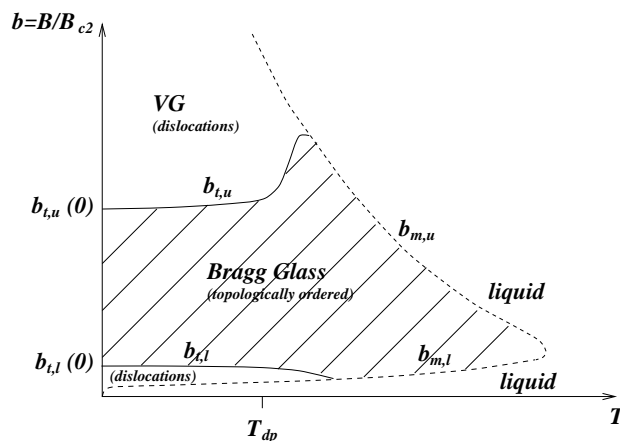


Fig. 1: Schematic phase diagram in the b - T plane ($b \equiv B/B_{c2}(T)$) showing the stability regime of the topologically ordered Bragg glass phase (hatched region). Its phase boundaries are given by the upper and lower branch $b_{t,u}(T)$ and $b_{t,l}(T)$ (solid lines) of a *topological* transition line, where dislocations proliferate. They terminate in the two branches $b_{m,u}(T)$ and $b_{m,l}(T)$ (dashed lines) of the *melting* curve, where the FL array melts into a (disordered) FL liquid.

II. POSITIONAL CORRELATION LENGTH R_L

To relate R_l to the microscopic parameters of the HTSC and the disorder strength, we have to review the crossover between the different pre-asymptotic regimes of the dislocation-free disordered FL array preceding the asymptotic Bragg glass phase, and the associated crossover length scales [1]. These crossovers are induced by the interplay between the FL interaction, the periodicity of the FL lattice and the disorder potential, which are in addition affected by thermal fluctuations, and lead to essentially two different pre-asymptotic regimes: On the shortest scales, we have the “Larkin” or “random force” regime of Larkin and Ovchinnikov [2], which crosses over to the so-called “random manifold” regime at the Larkin length, before the asymptotic Bragg glass behaviour sets in on the largest scales exceeding the positional correlation length. In between this sequence of crossovers, one additional length scale is set by the FL interaction, which describes a crossover from a “single vortex” behaviour to a “collective” behaviour.

In the following, we consider the usual experimental situation $\mathbf{H} \parallel \mathbf{c}$ of a magnetic field perpendicular to the CuO-planes of the HTSC. FL positions are parameterized by the two-component displacement-field $\mathbf{u}(\mathbf{R}, z) = \mathbf{u}(\mathbf{r})$ in a continuum approximation of the Abrikosov lattice, where \mathbf{R} is the vectors in the \mathbf{ab} -plane and z is the coordinate in the \mathbf{c} -direction, or by the Fourier transform $\tilde{\mathbf{u}}(\mathbf{K}, k_z) = \tilde{\mathbf{u}}(\mathbf{k})$. Let us adopt the convention to denote scales longitudinal

to the FLs in the z -direction by L and transversal scales in the \mathbf{ab} -plane by R . Moreover, it turns out to be convenient to use the *reduced induction* $b \equiv B/B_{c2}(T) = 2\pi\xi_{ab}^2/l^2$ to measure the strength of the magnetic field.

A. Interaction-induced length scale L^*

The dislocation- and disorder-free FL array can be described by elasticity theory (see Ref. [1] for a review) in the displacement field \mathbf{u} with the elastic moduli c_{11} , c_{44} and c_{66} , which can in general be dispersive (i.e., k -dependent in Fourier-space) due to the non-locality of the FL interaction. Except for extremely low magnetic fields, the FL lattice is essentially incompressible ($c_{11} \gg c_{66}$), and we can neglect longitudinal compression modes to a good approximation. Note also, that the shear modulus c_{66} is non-dispersive, because volume-preserving shear modes are not affected by the non-locality in the FL interaction. Then, the elastic Hamiltonian in the remaining transversal part $\tilde{\mathbf{u}}_T$ of the displacement field is of the form:

$$\mathcal{H}_{el}[\tilde{\mathbf{u}}_T] = \frac{1}{2} \int \frac{d^2\mathbf{K}}{(2\pi)^2} \frac{dk_z}{2\pi} \{c_{66}(\mathbf{K} \times \tilde{\mathbf{u}}_T)^2 + c_{44}[K](k_z \tilde{\mathbf{u}}_T)^2\} \quad (2)$$

The tilt modulus $c_{44} = c_{44}[K]$ is dispersive with

$$c_{44}[K] \simeq \hat{c}_{44} \frac{1}{1 + K^2 \tilde{\lambda}_c^2} . \quad (3)$$

The length scale for the onset of dispersion is

$$\tilde{\lambda}_c \equiv \lambda_c / (1 - b)^{1/2} , \quad (4)$$

because elements of tilted FLs lying in the ab -plane will start to interact on scales $R \lesssim \lambda_c$, where λ_c has to be modified at higher magnetic fields to the effective $\tilde{\lambda}_c$ in non-local elasticity theory [22]. As length scale for the onset of dispersion, $\tilde{\lambda}_c$ occurs as well in the Lindemann criterion (1). At $R \simeq l$, the result (3) crosses over to the *single vortex tilt modulus* c_{44}^s :

$$c_{44}^s \approx l^2 c_{44}[K \simeq 2\pi/l] . \quad (5)$$

From (2) we can obtain a scaling relation between scales L longitudinal to the FLs and transversal scales R for typical fluctuations involving elastic deformation:

$$L \sim R \left(\frac{c_{44}[2\pi/R]}{c_{66}} \right)^{1/2} . \quad (6)$$

The three-dimensional elastic Hamiltonian (2) is valid only on scales $R \gtrsim l$ or

$$L \gtrsim L^* \simeq \left(\frac{c_{44}^s}{c_{66}} \right)^{1/2} l \quad (7)$$

because of (6,5). When we consider fluctuations on scales $L \lesssim L^*$, the FL array breaks up into single FLs described by 1-dimensional elasticity in the longitudinal coordinate z , and the effects of FL interaction contained in c_{66} become irrelevant because transversal fluctuations on scales $R \lesssim l$ are not possible. Thus the interaction-induced length scale L^* separates a regime of ‘‘collective’’ behaviour described by 3D elasticity from a ‘‘single vortex’’ behaviour described by 1D elasticity. L^* starts to increase exponentially for $l/\lambda_{ab} = (b/2\pi)^{-1/2}/\kappa \gg 1$ in the dilute limit, (with $\kappa = \lambda_{ab}/\xi_{ab} \approx 100$ for BSCCO [1]) due to the exponential decay of $c_{66} \propto \exp(-l/\lambda_{ab})$. For the length scale L^* given by (7), the following interpolation formula can be obtained from the results in Refs. [1,2]:

$$L^* \approx \epsilon l f_\kappa \left(\frac{b}{2\pi} \right) \quad (8)$$

$$f_\kappa(x) = \begin{cases} x < \kappa^{-2} : 1 \\ x > \kappa^{-2} : (x\kappa^2)^{3/8} \exp\left(\frac{(x\kappa^2)^{-1/2}-1}{2}\right), \end{cases} \quad (9)$$

where $\epsilon = \lambda_{ab}/\lambda_c$ is the anisotropy ratio of the HTSC and $\epsilon \approx 1/100$ for BSCCO [1].

B. Larkin length L_ξ

When point disorder is introduced, every vortex at position \mathbf{R}_ν in the Abrikosov lattice experiences a pinning potential $V(\mathbf{r})$ with mean zero and short-range correlations

$$\overline{V(\mathbf{r})V(\mathbf{r}')} = \gamma\xi_{ab}^4\delta_\xi(\mathbf{r} - \mathbf{r}') , \quad (10)$$

where the overbar denotes an average over the quenched disorder. The strength of the disorder potential is given by $\gamma = n_{pin}f_{pin}^2$, where n_{pin} is the density of pinning centers and f_{pin} the maximum pinning force exerted by one pinning center, and the range of the disorder potential is given by the size ξ_{ab} of the core of a vortex. As proposed in Ref. [1], we introduce the basic energy (per length) scale $\epsilon_0 = (\phi_0/4\pi\lambda_{ab})^2$ and the dimensionless disorder strength δ as

$$\delta = \frac{\gamma\xi_{ab}^3}{(\epsilon_0\xi_{ab})^2} . \quad (11)$$

The interaction with the disorder is described by the Hamiltonian

$$\mathcal{H}_{dis}[\mathbf{u}] = \sum_\nu \int dz V(\mathbf{R}_\nu + \mathbf{u}(\mathbf{R}_\nu, z), z) . \quad (12)$$

For mean square displacements

$$u(R, L) \equiv \overline{(\mathbf{u}(\mathbf{r} + (\mathbf{R}, L)) - \mathbf{u}(\mathbf{r}))^2}^{1/2} \quad (13)$$

smaller than the effective scale r_T for variations of the disorder potential V , the FLs explore only *one* minimum of the disorder potential and perturbation theory in the displacements is valid. Expanding in (12) the disorder potential V in \mathbf{u} yields the *random force* theory of Larkin and Ovchinnikov [2]. At low temperatures, r_T is equal to ξ_{ab} but is increased by thermal fluctuations, and the (longitudinal) *Larkin length* L_ξ is defined as the crossover scale for the random force regime, at which the average FL displacement becomes of order of the effective range r_T of the point disorder:

$$u(L_\xi) \simeq r_T \approx \max \left\{ \xi_{ab}, \langle u^2 \rangle_{th}^{1/2}(L_\xi) \right\} \quad (14)$$

($\langle \dots \rangle_{th}$ denotes a purely thermal average at $V = 0$). It is important to note that for HTSCs such as BSCCO, the generic disorder strength is such that

$$L_\xi \lesssim L^* \quad (15)$$

and the random force regime lies entirely in the single vortex regime defined above. Therefore the Larkin length L_ξ is given by the single vortex result, which is at low temperatures [1]

$$L_\xi(0) \simeq \varepsilon \left(\frac{(\varepsilon\epsilon_0\xi_{ab})^2}{\varepsilon\gamma} \right)^{1/3} \simeq \varepsilon\xi_{ab} \left(\frac{\delta}{\varepsilon} \right)^{-1/3} . \quad (16)$$

This result holds as long as $r_T \simeq \xi_{ab}$. However, above the *depinning temperature* T_{dp} , r_T grows beyond ξ_{ab} [1]:

$$r_T^2 \simeq \xi_{ab}^2 \exp \left((T/T_{dp})^3 \right) , \quad (17)$$

where the depinning temperature T_{dp} is given by [1]

$$T_{dp}^s \simeq \varepsilon\epsilon_0\xi_{ab} \frac{\varepsilon\xi_{ab}}{L_\xi(0)} \simeq \varepsilon\epsilon_0\xi_{ab} \left(\frac{\delta}{\varepsilon} \right)^{1/3} . \quad (18)$$

Above T_{dp} , $L_\xi(T)$ increases exponentially with temperature due to the fact that random forces are only marginally relevant for a single FL with two-component displacements [1]:

$$L_\xi(T) \simeq L_\xi(0) \begin{cases} T \ll T_{dp} : & 1 \\ T > T_{dp} : & (T/T_{dp})^{-1} \exp \left((T/T_{dp})^3 \right) \end{cases} \quad (19)$$

Let us discuss estimates of the quantities L_ξ and T_{dp} at this point, which provide alternative measures of the disorder strength for BSCCO. As estimates in BSCCO, we use $\xi_{ab} \approx 20\text{\AA}$, $\epsilon_0 \xi_{ab} \approx 1000\text{K}$, $\epsilon \approx 1/100$, which is in accordance with Refs. [1,19]. In Ref. [19], the disorder strength is given by $T_{dp} \approx 10\text{K}$, which leads to $\delta/\epsilon \approx 1$ with (18). This estimate is considerably higher than typical values given in Ref. [1] for weak pinning. We will adopt these estimates of Ref. [19], which may apply for the relatively strong generic pinning in BSCCO. This yields very small values of the order of $L_\xi(0) \approx \epsilon \xi_{ab} \approx 0.2\text{\AA}$ for the (longitudinal) Larkin length in BSCCO (clearly, $L_\xi(0) \approx \epsilon \xi_{ab} \ll \epsilon l \lesssim L^*$ fulfills (15) and is indeed in the single vortex regime).

C. Positional Correlation Length R_l

On scales exceeding the Larkin length L_ξ the FLs start to explore many minima of the disorder potential V . However, as long as $u(R, L)$ is smaller than the FL spacing l , FLs are *not* competing for the same minima, and the FLs experience effectively *independent* disorder potentials. This leads to the approximation $\mathcal{H}_{dis}[\mathbf{u}] \approx \int d^3\mathbf{r} \tilde{V}(\mathbf{r}, \mathbf{u}(\mathbf{r}))$, where \tilde{V} has also short-range correlations in \mathbf{u} . This regime is referred to as *random manifold* regime [17]. For a d -dimensional (dispersion-free) elastic manifold with a n -component displacement field \mathbf{u} , the scaling behaviour of the $\langle uu \rangle$ -correlations is known to be

$$u(L) \sim L^{\zeta(d,n)} \quad (20)$$

with a roughness exponent $\zeta(d,n)$. We are interested here in the case $d = 1, n = 2$, which is realized on scales $L_\xi \lesssim L \lesssim L^*$ in the single vortex regime, where the FLs are described as 1-dimensional elastic manifolds, and the case $d = 3, n = 2$ on scales $L^* \lesssim L \lesssim L_l$ (or transversal scales $l \lesssim R \lesssim R_l$) in the collective regime, where the FL array is described as 3-dimensional elastic manifold. L_l and R_l are the *positional correlation lengths*, which are defined as the crossover scales for the random manifold regime, at which the average FL displacement becomes of the order of the FL distance l :

$$u(R_l, L_l) \simeq l. \quad (21)$$

On scales $R \gtrsim R_l$, where $u(R) \gtrsim l$, FLs start to compete for the *same* minima, and the periodicity of the FL lattice becomes crucial [5,17]. The FL array reaches its asymptotic behaviour of the Bragg glass phase with only logarithmically diverging $\langle uu \rangle$ -correlations, i.e., quasi long range positional order. The best available estimates for the roughness exponents are [23]

$$\zeta(1,2) \approx 5/8 \quad \text{and} \quad \zeta \equiv \zeta(3,2) \approx 1/5, \quad (22)$$

where the latter occurs also in the above Lindemann criterion (1). In the collective regime the scaling relation (20) gets slightly modified by the dispersion (3) of c_{44} to

$$u^2(R) \sim \left(\tilde{\lambda}_c^2 + R^2 \right)^{\zeta(3,2)}, \quad (23)$$

as can be checked by means of a simple Flory-type argument, where we equate the typical disorder energy and elastic energy (2) on *one* dominant scale.

The scaling relations (20,23) enable us to obtain the relation between the (transversal) positional correlation length R_l and the (longitudinal) Larkin length L_ξ , which will allow us to express R_l in terms of microscopic parameters. Applying the scaling relation (20) for the $\langle uu \rangle$ -correlations to the single vortex random manifold regime on longitudinal scales $L_\xi \lesssim L \lesssim L^*$, we obtain

$$u_* \equiv u(R = l, L = L^*) \simeq r_T \left(\frac{L^*}{L_\xi(T)} \right)^{\zeta(1,2)}. \quad (24)$$

In the same manner we can use (23) in the collective random manifold regime on transversal scales $l \lesssim R \lesssim R_l$:

$$l^2 \simeq u^2(R_l) \simeq u_*^2 \left(\frac{\tilde{\lambda}_c^2 + R_l^2}{\tilde{\lambda}_c^2 + l^2} \right)^{\zeta(3,2)}. \quad (25)$$

Using (24,25), R_l can be expressed as

$$R_l(T) \simeq (\tilde{\lambda}_c^2 + l^2)^{1/2} \left(\frac{l}{r_T} \right)^{1/\zeta(3,2)} \left(\frac{L_\xi(T)}{L^*} \right)^{\zeta(1,2)/\zeta(3,2)}. \quad (26)$$

With the results (17) for r_T , (8) for L^* , and (19) for $L_\xi(T)$ together with $\zeta(3,2) \approx 1/5$ and $\zeta(1,2) \approx 5/8$ (22), this yields the desired expression for R_l :

$$R_l(0) \approx (\tilde{\lambda}_c^2 + l^2)^{\frac{1}{2}} \left(\frac{b}{2\pi} \right)^{-15/16} \left(f_\kappa \left(\frac{b}{2\pi} \right) \right)^{-25/8} \left(\frac{\delta}{\varepsilon} \right)^{-25/24}$$

$$R_l(T) \approx R_l(0) \begin{cases} T \ll T_{dp} : 1 \\ T > T_{dp} : (T/T_{dp})^{-25/8} \exp\left(\frac{5}{8} (T/T_{dp})^3\right) \end{cases} \quad (27)$$

For inductions $b = 10^{-4} \dots 10^{-1}$ in the dense limit $b \gtrsim 2\pi/\kappa^2$, we obtain with $\delta/\varepsilon \approx 1$ and $\lambda_c \approx 2 \cdot 10^5 \text{ \AA}$ (transversal) positional correlation lengths $R_l(0) \approx (10^5 \dots 10^2) \cdot \lambda_c \approx 2 \cdot (1 \dots 10^{-3}) \text{ cm}$, which is extremely large and indicates that over a wide range of length scales the pre-asymptotic random manifold regimes should be observable rather than the asymptotic Bragg glass regime.

III. LINDEMANN CRITERION

Let us now show the equivalence of the Lindemann-like criterion (1) derived in Ref. [18] to the conventional form of the Lindemann criterion generalized to a disordered system. The Lindemann criterion has been proven as a very efficient phenomenological tool to obtain the thermal melting curves of lattices, e.g. the disorder-free FL lattice. There, it is formulated in its conventional form

$$\langle u^2 \rangle_{th} = c_L^2 l^2, \quad (28)$$

with a Lindemann-number $c_L \approx 0.1 - 0.2$. For the thermal melting of the FL array, the main contributions to the left hand side of (28) come from fluctuations on the *shortest* scale, which is in the transverse direction the FL spacing l , i.e., $\langle u^2 \rangle_{th} \approx \langle u^2 \rangle_{th}(R = l)$ (note that we apply again a convention like (13)). The straightforward generalization of (28) to the disorder-induced melting by dislocations is therefore

$$\overline{\langle u^2 \rangle}(R = l, L = L^*) \equiv u_*^2 = c_L^2 l^2, \quad (29)$$

where we consider again fluctuations on the shortest scale $R \simeq l$. In Ref. [18], one derivation of the criterion (1) was based on a variational calculation for a layered superconductor in a parallel field. There it was found, that unbound dislocations proliferate indeed on the shortest scale at the topological transition described by (1), i.e., in between every layer and thus with a distance l . This suggests that the use of the conventional phenomenological Lindemann criterion in the form (29) might be one possibility to obtain the topological transition line.

This can be further justified by showing that the criterion (1), which was *derived* on the bases of a variational calculation and a scaling argument in Ref. [18], is actually *equivalent* to the phenomenological Lindemann criterion (29). Considering the relation (25) between u_* and l , it becomes immediately clear that (1) is the analog of the Lindemann criterion (29) formulated in terms of the underlying transversal scales rather than the corresponding displacements: Using (25), the criterion (1) for the stability of the Bragg glass becomes equivalent to

$$u_*^2 \lesssim c^{-1} l^2. \quad (30)$$

Furthermore, we can identify

$$c \approx c_L^{-2}. \quad (31)$$

We see that the equivalence of the criterion (1) to the phenomenological Lindemann criterion (29) includes the agreement of the appearing numerical factors. The value for the Lindemann-number $c_L \approx 0.15$, widely used in the literature, produces a good agreement in (31) with the value $c \approx \mathcal{O}(50)$ obtained by the variational calculation. This equivalence to a scenario where disorder-induced fluctuations on the shortest scale “melt” the FL array favours a first order transition scenario for the topological transition, which could not be excluded in the experiments [13]. As we will see, the quantity u_*^2 is equivalent to the mean square displacement of the “effective” FL studied in the “cage model” of Ertaş and Nelson [19]. They apply the Lindemann criterion directly in its phenomenological form (29) to the “caged” FL. Using (31,22), we can cast the Lindemann-like criterion (1) into the form

$$R_l > c_L^{-1/\zeta} \left(l^2 + \tilde{\lambda}_c^2 \right)^{1/2} \approx c_L^{-5} \max\{l, \lambda\}. \quad (32)$$

IV. PHASE DIAGRAM

Let us now address the issue of phase boundaries of the topologically ordered Bragg glass in the B-T plane as they follow from the Lindemann-like criterion (1) in the above form (32). The results are summarized in Fig. 1. The boundary of the regime given by (32) defines a *topological transition line* $B_t(T)$, where dislocations proliferate and the topological order of the Bragg glass phase is lost. The upper branch $b_{t,u}(T)$ of this line can be obtained by applying the expression (27) for the positional correlation length R_l in the dense limit $b \gtrsim 2\pi/\kappa^2$ (i.e. $f_\kappa \equiv 1$ in (27)) to the criterion (32), which finally yields a condition $b < b_{t,u}(T)$ in the b-T plane with

$$b_{t,u}(0) \approx 2\pi \left(\frac{\delta}{\varepsilon}\right)^{-10/9} c_L^{16/3} \approx 2\pi \left(\frac{\varepsilon\varepsilon_0\xi_{ab}}{T_{dp}^s}\right)^{10/3} c_L^{16/3}$$

$$b_{t,u}(T) \approx b_{B,u}(0) \begin{cases} T \ll T_{dp} : 1 \\ T > T_{dp} : (T/T_{dp})^{-10/3} \exp\left(\frac{2}{3}(T/T_{dp})^3\right). \end{cases} \quad (33)$$

Note that the transition line (33) is identical to the one obtained by Ertas and Nelson [19] by applying the conventional phenomenological Lindemann criterion to a ‘‘cage model’’ for a single FL (this demonstrates the equivalence of the displacement u_* as defined in (29) to the average displacement of the ‘‘caged’’ FL). The estimates $c_L \approx 0.17$ and $\delta/\varepsilon \approx 1$ lead to $b_{t,u}(0) \approx 5 \cdot 10^{-4}$ or $B_{t,u}(0) \approx 400\text{G}$ [19], which is in good agreement with the experiments [12,13] as well as the overall transition line shape (see Fig. 1) [13]. For stronger anisotropy or effectively larger disorder strength δ/ε the transition line $b_{t,u}$ decreases in magnetic field, and the stability region of the topologically ordered Bragg glass shrinks in accordance with the experimental findings in Ref. [13]. For temperature $T < T_{dp}$ the transition line is essentially temperature-independent because the mechanism for the proliferation of dislocation loops is purely disorder-driven at low temperatures [18]. For $T > T_{dp}$ it increases exponentially due to the very effective weakening of the pinning effects by thermal fluctuations in the single vortex regime, and at some temperature slightly above T_{dp} , the transition line will terminate in the upper branch of the melting curve $b_{m,u}(T)$, which is

$$b_{m,u}(T) \approx 30 \frac{(\varepsilon\varepsilon_0\xi_{ab})^2 c_L^4}{T^2} \quad (34)$$

in this regime [1]. Beyond the melting curve $b_{m,u}(T)$, the FL array melts into a disordered FL liquid, and the Bragg glass order is destroyed by the thermal fluctuations, whereas above the transition line $b_{t,u}(T)$ the Bragg glass ‘‘melts’’ by disorder-induced fluctuations, when unbound dislocation loops proliferate. As noted in [19], $b_{t,u}(T)$ is well below the so-called ‘‘decoupling field’’ beyond which the layered structure of the HTSC requires a discrete description in the **c**-direction.

At low inductions in the dilute limit $b \ll 2\pi/\kappa^2$, the criterion (32) will be violated due to the exponential decrease of the shear modulus c_{66} , or increase of the interaction-induced length scale L^* (8) encoded in the function f_κ (9). At low temperatures $T \lesssim T_{dp}$, the temperature independent lower branch of the topological transition line $b_{t,l}(T) \approx b_{t,l}(0)$ can be determined as the smaller of the two solutions of

$$\left(f_\kappa\left(\frac{b_{t,l}(0)}{2\pi}\right)\right)^{10/3} \frac{b_{t,l}(0)}{2\pi} \approx \left(\frac{\delta}{\varepsilon}\right)^{-10/9} c_L^{16/3}. \quad (35)$$

At temperatures $T \gg T_{dp}$ well above the depinning temperature, the asymptotics

$$b_{t,l}(T) \sim \frac{25\pi}{2\kappa^2} \left(\frac{T}{T_{dp}}\right)^{-6} \quad (36)$$

is obtained. Thus, $b_{t,l}(T)$ will terminate in the lower branch of the melting curve $b_{m,l}(T)$, which increases logarithmically with temperature [1]

$$b_{m,l}(T) \approx \frac{2\pi}{\kappa^2} \ln^{-2} \left(\frac{(\varepsilon\varepsilon_0\xi_{ab})^2 c_L^4 \kappa^2}{T^2}\right). \quad (37)$$

With $c_L \approx 0.17$ and $\delta/\varepsilon \approx 1$, we can obtain from (35) numerically $b_{t,l}(0) \approx 0.03(2\pi/\kappa^2) \approx 2 \cdot 10^{-5}$, which is by a factor of 25 smaller than $b_{t,u}(0)$ and experimentally hard to verify due to the small inductions $B_{t,u}(0) \approx 16\text{G}$. From (35) it is clear that the transition line $b_{t,u}(T)$ increases with the disorder strength so that the stability region of the topologically ordered Bragg glass shrinks.

V. CONCLUSION

In conclusion, we have obtained the region in the phase diagram of BSCCO in the B-T plane, where the topologically ordered vortex glass should be observable, and the topological transition lines $B_{t,u}(T)$ and $B_{t,l}(T)$, where dislocation loops proliferate. The resulting phase diagram, as given by the formulae (33,35,36) and depicted in Fig. 1, is in reasonable agreement with the experimental data of Refs. [12,13] as well as the simulation data of Ref. [21]. The phase diagram is based on the Lindemann criterion (1,32), which has been *derived* using a variational calculation and a scaling argument in Ref. [18]. We have demonstrated the equivalence to the conventional phenomenological formulation of the Lindemann criterion (29) up to the involved numerical factors, i.e., the Lindemann-number c_L . Our results for the upper branch of the topological transition line $B_{t,u}(T)$ agree with Ref. [19], where the conventional phenomenological Lindemann-criterion was applied to the disorder-induced “melting” in the framework of a “cage model”.

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