Interacting Arrays of Lines and Steps in Random Media

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The phase diagram of two interacting planar arrays of directed lines in random media is obtained by a renormalization group analysis. The results are presented in the contexts of the roughening of anisotropically reconstructed crystal surfaces, and the pinning of vortex line arrays in planar Josephson junctions. Among the findings are the stability of a flat anisotropically reconstructed surface, a novel second-order phase transition with continuously varying critical exponents, and the generic disappearance of the glassy "superrough" phases found previously for a single line array. Relevance of our results to the issue of replica-symmetry breaking is also discussed. [S0031-9007(96)01639-0]

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The statistical mechanics of planar arrays of directed lines is of interest to various physical systems. For example, the steps formed on a vicinal crystal surface can be modeled by an array of directed lines confined in a plane [1]. The same model describes Josephson vortex lines in a planar Josephson junction subject to a parallel magnetic field [2,3]. An important issue concerns the behavior of such arrays of lines in the presence of quenched disorders. It is known that the lines are pinned by point disorders and become "glassy" at low temperature [4]. However, different analytic [4–8] and numeric [9] studies have yielded conflicting results regarding the nature of the glass phase. A subject of debate is the possible breaking of replica-symmetry in the glass phase [10,11].

In this article we study the effect of point disorders on *two* interacting species of lines in a plane. This problem arises in the study of anisotropically (2×1) reconstructed gold (110) surfaces [12], where two kinds of (3×1) microfacets can be treated as two species of interacting lines (Fig. 1). Previous studies of the pure system have revealed a rich phase diagram with a variety of phases as a function of the interaction parameters [12–14]. The inclusion of point disorders, say crystalline defects originating from a disordered substrate, induces deformations in the trajectories of the microfacets. Similar issues arise in two layers of magnetically interacting Josephson vortex lines. Performing a renormalization group (RG) analysis in replica space, we are able to obtain a detailed picture of the RG flow. The result is applied to discuss the phase diagram of the reconstructed surfaces as well as the structure of the glass phases obtained for the vortex arrays.

A single species of directed lines confined in a plane containing quenched randomness can be described by the continuum Hamiltonian [2,3]

$$\beta \mathcal{H}_{2\mathrm{D}}[\phi, V] = \int_{\mathbf{r}} \left\{ \frac{K}{2} (\nabla \phi)^2 - V(\mathbf{r}) \rho(\phi(r), \mathbf{r}) \right\} \quad (1)$$

on length scales exceeding the line spacing l. The first part of (1) gives the elastic energy of the line array in terms of a displacement-like scalar field $\phi(\mathbf{r})$ (a displacement by l corresponds to a shift of 2π in ϕ), characterized by an (isotropized) elastic constant K [15]. The second term describes density variations $\rho(\phi, \mathbf{r})$ induced by a random potential $V(\mathbf{r})$, reflecting the attraction of the line defects [e.g., the (3×1) microfacets or the vortex lines] by the quenched point defects in the background. The density field has the form $\rho[\phi(\mathbf{r}), \mathbf{r}] \approx \rho_0[1 - \partial_x \phi / 2\pi\rho_0 + 2\cos(2\pi\rho_0 x - \phi)]$, where $\rho_0 = 1/l$ is the average line density, and $\mathbf{r} = (x, z)$ with z along the line direction. The random potential is taken to have zero mean with short-range correlations $\overline{V(\mathbf{r})V(\mathbf{r}')} = g\,\delta(\mathbf{r} - \mathbf{r}')$ of (bare) strength g.

An interaction between two such species of lines in the form of $\int_{\mathbf{r}_1, \mathbf{r}_2} V_{\mathrm{int}}(\mathbf{r}_1 - \mathbf{r}_2) \rho(\phi_1, \mathbf{r}_1) \rho(\phi_2, \mathbf{r}_2)$ with a short-ranged potential V_{int} leads to

$$\beta \mathcal{H}_{\mathrm{int}} \approx \int_{\mathbf{r}} \{ 2\mu\rho_0^2 \cos(\phi_1 - \phi_2) + K_\mu \nabla\phi_1 \cdot \nabla\phi_2 \}, \quad (2)$$

with $\mu = \int_{\mathbf{r}} V_{\mathrm{int}}(\mathbf{r})$ and $K_\mu = \mu / 8\pi^2$ [15]. We assume the disorder potential V_i acting on species i to be statistically identical, with the cross-correlations $\overline{V_1(\mathbf{r})V_2(\mathbf{r}')} = g_\mu \delta(\mathbf{r} - \mathbf{r}')$ to be specified below. The full Hamiltonian of our system, $\mathcal{H}[\phi_1, \phi_2] = \sum_{i=1}^{2} \mathcal{H}_{2D}[\phi_i, V_i] + \mathcal{H}_{\mathrm{int}}[\phi_1, \phi_2]$, can then be written in a succinct form (after neglecting irrelevant terms [15]),

$$\beta \mathcal{H} = \int_{\mathbf{r}} \left\{ \frac{K_{ij}}{2} \nabla\phi_i \cdot \nabla\phi_j - \mathbf{w}_i \cdot \nabla\phi_i - W_i(\phi_i, \mathbf{r}) + 2\mu\rho_0^2 \cos(\phi_1 - \phi_2) \right\}, \quad (3)$$

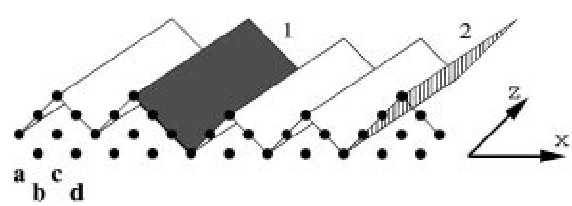

FIG. 1. Two kinds of (3×1) microfacets on a (2×1) reconstructed crystal surface. The background (2×1) facets can be on four possible sublattices (marked "a"–"d"). Each (3×1) facet shifts the phase by one sublattice.

with effective random potentials W_i and $\mathbf{w}_i$ whose correlators are $\overline{W_i(\phi,\mathbf{r})W_j(\phi',\mathbf{r}')} = 2g_{ij}\rho_0^2\cos(\phi - \phi')\,\delta(\mathbf{r}-\mathbf{r}')$, $\overline{\mathbf{w}_i(\mathbf{r})\mathbf{w}_j(\mathbf{r}')} = \Delta_{ij}\delta(\mathbf{r}-\mathbf{r}')$, with $\Delta_{ij} = g_{ij}/8\pi^2$. The parameters of the theory are $K_{ij} = K, g_{ij} = g, \Delta_{ij} = \Delta$ for $i = j$, and $K_{ij} = K_\mu, g_{ij} = g_\mu, \Delta_{ij} = \Delta_\mu$ for $i \neq j$. The cosine couplings reflects the discrete nature of the lines.

A physical observable of interest for the crystal surface is the height profile $h(\mathbf{r})$ of the surface. It is given by $h(\mathbf{r}) = |\phi_1(\mathbf{r}) - \phi_2(\mathbf{r})|/2\pi$ since lines from species 1 and 2 represent upward and downward $|3 \times 1|$ microfacets, respectively (Fig. 1), and it is the *difference* of the two that determines the height profile [14]. Note also that the (2×1) reconstructed surface has a choice of *four* possible sublattices on the surface, as marked in Fig. 1. Each step is thus also a domain wall separating the domains of the sublattices. An order parameter capturing the ordering of the domains is $e^{i\varphi(\mathbf{r})}$, where $\varphi(\mathbf{r}) \equiv [\phi_1(\mathbf{r}) + \phi_2(\mathbf{r})]/4$ specifies the domain phase. In the following analysis, we shall characterize the system by excluding *all* forms of topological defects in ϕ_1 and ϕ_2. While this approximation is reasonable for the Josephson junctions [16], it is not always valid for the reconstructed surfaces where vortices in the phase field φ can play an important role [12–14]. For the latter case, our results will be used to determine the relevancy of the vortices in the presence of quenched disorders.

To find the large scale behaviors of the system in the absence of topological defects, we apply the Replica method to average over disorders. A replica-symmetric RG analysis [17,18] yields the following recursion relations (to bilinear order) upon a change of scale of e^l:

$$\begin{aligned} dg/dl &= (\kappa - \tau)g - g^2 - g_\mu \mu\,, \\ dg_\mu/dl &= (\kappa - \tau - \delta)g_\mu - g g_\mu - g\mu\,, \\ d\mu/dl &= (2\kappa - \delta)\mu - (\tau + \kappa)g_\mu - g\mu\,, \\ d\kappa/dl &= \mu(\mu - 2g_\mu)/2, \\ d\delta/dl &= (g^2 - g_\mu^2)/2\,. \end{aligned} \tag{4}$$

Here $\kappa = 1 - [4\pi(K - K_\mu)]^{-1}$ is a reduced elasticity parameter $\delta = 8\pi(\Delta - \Delta_\mu)$, and a nonuniversal numerical factor (4π for our IR-regularization) is absorbed into g, μ, and g_μ. The RG flow is controlled by the reduced temperature $\tau = [4\pi(K + K_\mu)]^{-1} - 1$, which is not renormalized due to a statistical tilt symmetry. There exists a sixth RG equation $d\overline{\delta}/dl = (g^2 + g_\mu^2)/2$ for the parameter $\overline{\delta} = 8\pi(\Delta + \Delta_\mu)$. While $\overline{\delta}$ does not feedback into (4), its flow controls the scaling of the phase field φ and will be crucial in determining the relevancy of vortices in φ.

Before delving into the structure of the RG flow, we first mention two limiting subproblems which have been studied previously. In the limit μ, $g_\mu = 0$, the elasticity parameter κ is also not renormalized, and the RG equation has the same structure as that obtained for a single species of lines [4] with an effective temperature $\tau - \kappa$. As shown in Fig. 2 [inset (a)], the disorder (g) is irrelevant at high temperatures ($\tau > \kappa$), yielding the usual logarithmic roughness for 2D surfaces, accompanied by a quasi-long-ranged domain order. We refer to this as the decoupled line (DL) phase. At $\tau = \kappa$, the marginal irrelevance of g yields a marginally coupled line phase (ML), which again has logarithmic roughness and quasi-long-ranged domain ordering. At low temperatures ($\tau < \kappa$), the disorder is relevant. The resulting glass phases are described by the line of fixed points $g^*(\kappa) = \kappa - \tau$ which are perturbatively accessible for $|\kappa|, |\tau| \ll 1$. Since complete decoupling implies also [see Eq. (2)] that $K_\mu = 0$ or $\kappa + \tau = 0$, only the point $g^* = 2|\tau|$ (and $\mu^* = 0$, $\kappa^* = |\tau|$) along the line $g^*(\kappa)$ is the physical fixed point; it describes a decoupled glass (DG) phase. The surface is *super rough* [1] in the DG phase, with $\overline{\langle h^2 \rangle} \sim \ln^2 L$ on large scales L. The glassiness is also reflected by a disordered (short-ranged) domain order due to the anomalous scaling of the domain phase $\overline{\langle \varphi^2 \rangle} \sim \ln^2 L$. The logarithmic singularities in $\overline{\langle h^2 \rangle}$ and $\overline{\langle \varphi^2 \rangle}$ both result from the divergence of $\overline{\delta}$ when g^* is finite.

Another well known limit of our problem is that of vanishing disorder ($g, g_\mu = 0$), where a Kosterlitz-Thouless (KT) transition occurs independent of τ [see Fig. 2, inset (b)]. For large coupling $|\mu|$, the two species become locked together, forming elastically coupled line (EL) phases with $|\mu^*|, \kappa^* \longrightarrow O(1)$. Since the up and down steps are now paired, in phase ($\phi_1 = \phi_2$) for $\mu > 0$ or out of phase ($\phi_1 = \phi_2 + \pi$) for $\mu < 0$, the surface is *flat*, with quasi-long-ranged domain order. The issue of vortices in the phase field φ has been addressed in Ref. [14]. The vortices are equivalent to *loops* involving the intersection of the four types of domain walls. The relevance of the vortices is controlled by the scaling of $\overline{\langle \varphi\varphi \rangle}$, which depends only on τ in the pure problem. Simple power counting along the line of Ref. [14] indicates that the vortices are relevant if $\tau > 0$ and irrelevant if $\tau < 0$. In the presence of quenched disorders, it naively appears that the vortices might be relevant in the low temperature regime ($\tau < 0$) as well, due to the anomalous variations in φ induced by the disorders. This is, however, not always the case as the following analysis will show.

We shall focus on two choices of disorder which are of particular interest: (i) identical disorder for the two species ($g_\mu = g$, $\delta = 0$) and (ii) completely uncorrelated disorder ($g_\mu = 0$). Case (i) is the generic situation for steps on the anisotropically reconstructed surfaces with disorder. It can also be specifically constructed for the two layers of Josephson vortex lines. The most striking feature of the RG flow in the low temperature ($\tau < 0$) regime [Fig. 2] is the strong instability of the DG fixed point with respect to interspecies interaction $\mu \neq 0$. An attractive interaction ($\mu < 0$) favors the two species to lock into the same configuration, i.e., $\phi_1 = \phi_2$ or $h = 0$ (flat). Once locked, the system acts effectively as a single species with a doubled elastic constant K, or equivalently a lower effective temperature, so that the effective single species problem is in the glass phase. Correspondingly, (4) yields (for all $\tau < 0$) an RG flow away from the unstable DG fixed point to a sink with strong interspecies coupling *and* strong disorder $[g^*, \kappa^*, -\mu^* \rightarrow O(1)]$. We refer to this as the elastically coupled glass (EG) phase. Because fluctuation in φ is large for both $\tau > 0$ (entropy driven) and $\tau < 0$ (disorder driven), vortices in φ are *always relevant* for attractive interspecies interactions. Proliferation of vortices (or loops of domain walls) restores the isotropy of the surface at large scales, rendering the anisotropic treatment meaningless. In the asymptotic isotropic phase, coupling to bulk disorder is likely to roughen the surface as in [1]. However, a detailed description in that regime is beyond the scope of this paper.

A repulsive interaction ($\mu > 0$) competes with fluctuations in the random potential, which still attempts to lock the two species into the same configuration. On the low temperature side ($\tau \lesssim 0$) [19], this competition leads to *two* RG sinks separated by a second-order phase transition: If the repulsive interaction dominates, the two species avoid each other by locking into a configuration with $\phi_1 = \phi_2 + \pi$, i.e., with one species displaced by half a line spacing with respect to the other. Such a configuration can be interpreted again as a single species, but now with a doubled line density. This leads to a *higher* effective temperature, such that the effective single species system is *not* glassy, with $g^* = 0$. Corresponding to this scenario, we find for weak bare disorder a RG flow away from DG towards the fixed point EL, and the RG trajectories approach their pendants in the disorder-free subproblem. For stronger disorders, however, we obtain a RG flow from DG towards the fixed point ML, since the disorder weakens the interspecies coupling μ and κ, while the coupling μ in turn weakens the disorder g. Note that both the EL and ML phases are *stable* to the formation of dislocations in the phase field φ at low temperatures [19], since $\overline{\delta}^*$ is finite when $g^* = 0$. The phase transition separating the EL phase and the ML phase is governed by an unstable fixed point (T) at $(\kappa^*, g^*, \mu^*) = (-1, 2, 4)|\tau|/7$, which is the attractor of the plane of separatrix $g \simeq \mu + 2\kappa$. The phase transition is second order for $\tau < 0$, with an algebraically diverging correlation length (characterized by an exponent $\nu = 2|\tau|/7$) upon crossing the separatrix. It is a remarkable feature of this system that while both RG sinks are (at least marginally) *disorder free*, the unstable fixed point governing the transition is *disorder dominated*.

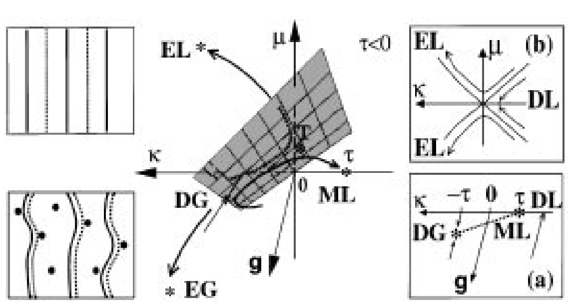

FIG. 2. RG flow for identical disorder ($g_\mu = g, \Delta = \Delta_\mu$). Insets (a) and (b) describe the flow for the subcase with no interspecies coupling ($\mu = 0$) and no disorders ($g = 0$), respectively.

The above analysis of the RG flow can be straightforwardly turned into a phase diagram. We define an interspecies interaction energy $U = \mu T$, and present the phase diagram in the (U, T) space, for the interesting $U \geq 0$ sector at a fixed disorder strength g [see Fig. 3(a)]. The superrough DG phases exist at $U = 0$ below a critical temperature T_c given by $K_c = 1/(4\pi)$. These phases are marked by the thick wavy line in Fig. 3(a) and are unstable to (disorder-generated) dislocations in φ. For $U > 0$, a separatrix $g = \mu + 2\kappa$ separates the flat, pure phase (EL) at low temperature and large repulsion from the two high temperature phases. At very high temperatures ($T \gg T_c$), the system is in the pure decoupled phase (DL) which is unstable to (thermally generated) dislocations in φ. Upon lowering the temperature beyond the line $\tau = 0$ (thin solid line), the system settles into the *stable* ML phase for weak repulsive interaction (compared to g). Further lowering the temperature beyond the separatrix (the thick solid line), the system makes a second-order transition from the ML to the EL phase which is also stable with respect to dislocations. Note that because the critical properties there are controlled by the fixed point T which depends on τ, the critical exponents governing this transition actually vary *continuously* along the thick solid line. The second-order transition terminates at a point where the separatrix intersects the line $\tau = 0$ [the open circle in Fig. 3(a)]. We expect the transition between DL and EL at higher temperatures to be the same as that of the disorder-free case [14].

Perhaps the most striking result of the above analysis is the suppression of glass order below T_c, by applying a small repulsion between the two species of lines. This effect stabilizes the anisotropy of the reconstruction and the flatness of the surface. It is also quite interesting from a more general theoretical perspective: The glass order was found previously for a single line array in random media by analysis based on the replica-symmetric RG method [1,4,5], but was not found in more recent studies using a variational method with replica-symmetry breaking (RSB) [7,8]. The latter finds instead the ML

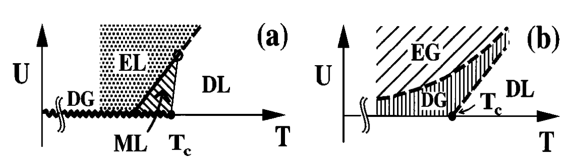

FIG. 3. Phase diagram for two species of lines in (a) identical and (b) uncorrelated random media. In (a), only the $U < 0$ sector corresponding to repulsive interactions is shown, while in (b), repulsive and attractive interactions lead to the same phase diagram (see text).

phase, which appears naively to be consistent with our findings here. However, our result can in fact be used to question the internal consistency of the RSB scheme: As described in Ref. [20], a physical way of probing the existence of RSB is to take two *physical* replicas of a system (in identical random potential) and monitor the response to a small repulsion between the replicas. If there is a degeneracy of low free energy states (which the RSB scheme attempts to describe), then an infinitesimal repulsion between the replicas will force the two to occupy different states which have *similar glassy properties* and little overlap. The system we have analyzed so far can be interpreted as two physical replicas in the same random potential. In our case, we see that a small repulsion has a much stronger effect in that it gives rise to qualitatively different behaviors, i.e., from DG to ML. This indicates that the glass order of a single line array is extremely fragile, making it quite different from the usual scenario expected of the usual stable glass phases described by zero temperature fixed points. Thus from the viewpoint of the replica-symmetric RG analysis, the absence of glass order from the solution of the variational treatment is not surprising, as it may be the result of subtle interactions introduced by the RSB scheme itself.

We continue with a short discussion of the case where the disorder potentials acting on the two species are uncorrelated, i.e., with the bare $g_\mu = 0$. This is the generic case for two layers of Josephson vortex lines in disordered planar Josephson junctions. Uncorrelated disorder tends to *decouple* the two species of lines and competes with the locking effect of the interspecies interaction. (Here attractive and repulsive interactions are qualitatively similar, up to a relative π-phase shift between the two species.) The RG analysis is more complicated than before because one has to consider the full set of equations in (4). [Note that g_μ is generated by μ and g.] The basic features of the phase diagram can be obtained by observing that the RG flow is dominated by two KT transitions which can be found in two subproblems of (4): (i) the KT transition of the disorder-free subproblem; (ii) the KT transition which occurs in the space (g_μ, δ), when $g = \mu = \kappa + \tau = 0$. The latter has a critical separatrix $g_\mu/\sqrt{2} = \delta + 2\tau$ which isolates two regions of flow to a sink with $g_\mu = 0$ and $\delta > 0$, and another sink where g_μ grows. The first sink is consistent with a decoupled glass (DG) phase as the eigenvalue for the flow of μ becomes negative there, while the second sink is consistent with the elastically coupled glass (EG) phase where $|g_\mu| \to g$. The form of this separatrix for the regime of physical initial conditions (e.g., $g, \mu \gg g_\mu$) is not known analytically. We determined it numerically to be of the form $\mu \simeq cg$, with the numerical constant $c \approx 0.5$. This condition, combined with subproblem (i) leads to the phase diagram depicted in Fig. 3(b). It is interesting to note that the condition $\mu \simeq cg$ separating the decoupled and coupled glass phases is very similar (including the numerical value of c) to the result obtained recently for a problem involving *many* layers of vortex lines [21].

In conclusion, we have presented a detailed RG analysis for a model of two interacting planar line arrays in random media. Among the findings are a novel second-order phase transition with continuously varying critical exponents, the stability of the anisotropic flat phase for repulsive interactions, and the replacement of the superrough glass phase by a marginally coupled phase.

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